Mth 076: Applied Geometry
(Individualized Sections)

MODULE FOUR STUDY GUIDE

INTRODUCTION TO GEOMETRY

Pick up Geometric Formula Sheet (This sheet may be used while testing)

Assignment Eleven: Problems Involving Circumference and Area of Circles

A. Read pages 51 & 52 in your textbook. Study examples 1-3 odd on page 52.
B. Work examples 2-4 even on page 52 and check your results.
C. **Assignment Eleven:** Do problems 2, 4, 8-28 even in Exercise 4, Objective A, pages 53, 54.

Assignment Twelve: Problems Involving Definitions and Properties of Circles

A. Read and study pages 1-3 in the Study Guide.
B. **Assignment Twelve:** Do problems 1-4 on pages 3 & 4 in the Study Guide.

Assignment Thirteen: Problems Involving Length of Arc and Area of a Sector

A. Read pages 5 & 6 in the Study Guide
B. Study examples 1-4 in the Study Guide.
C. **Assignment Thirteen:** Do problems 1-10 on pages 6-8 in the Study Guide.

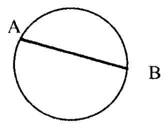
Review Assignment: Do the module four review assignment available in the math learning center and submit it to an instructor for grading.

Module Test: After successfully completing the three assignments and the review assignment, ask an instructor for a module four test.

Writing Assignment: Pick up several pieces of ½ inch grid paper which measures 7 inches by 9 inches and is available in the Math Learning Center. Create several rectangular troughs by cutting different size squares from the four corners of the paper. Calculate the volume of each trough and plot the length of square cut out vs. the volume of the trough on graph paper. From this graph, try to predict the length of the square to cut out that will result in the largest volume of the trough. Write an algebraic equation that represents the volume of the trough in terms of the length of the side of the square cut out. Use a graphing calculator available in the math learning center to graph this equation and find the length of the square to be cut out that will result in the largest volume of the trough. Compare this result with your prediction from your graph on graph paper. This paper is due before the module six test is taken.

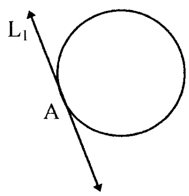
Circles: Some More Definitions and Properties

Chord: A line segment having its end points on the circle.



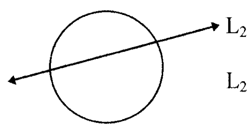
Segment AB is a chord

Tangent line: A line that touches the circle at exactly one point.



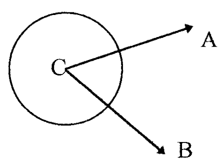
L_1 is a tangent line. Point A is the point of tangency.

Secant line: A line that passes through two points of the circle.



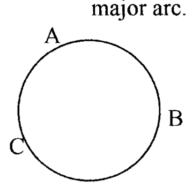
L_2 is a secant line.

Central Angle of a Circle: An angle with its vertex at the center of the circle.



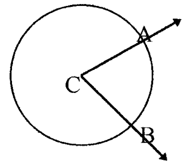
$\angle ACB$ is a central angle, given that point C is the center of the circle.

Arc of a Circle: The part of the circle between and containing two specified points on the circle. There are two such arcs on a circle, the minor arc and the major arc.



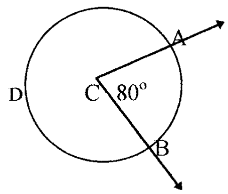
The portion of the circle from point A to point B is the minor arc AB. The portion of the circle from point A through point C to point B is the major arc ACB. Note that a minor arc is indicated with two points and a major arc is indicated with three points.

Intercepted Arc: The arc of a circle between the two rays of an angle.



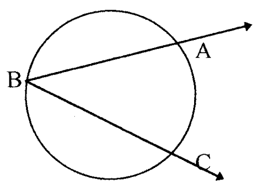
∠ ACB intercepts arc AB.

Measure of the Arc of a Circle: An arc is measured by its central angle. The measure of an arc of a circle and the central angle that intercepts the arc are equal.



The measure of the minor arc AB is 80° since the measure of the central angle ACB is 80°. The measure of the major arc ADB is 280° since $360^\circ - 80^\circ = 280^\circ$.

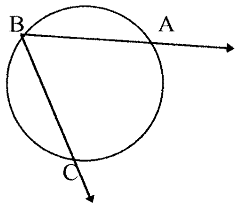
Inscribed Angle: An angle is inscribed in an arc if the sides of the angle contain the end points of the arc and the vertex of the angle is a point on the arc. (Not an end point)



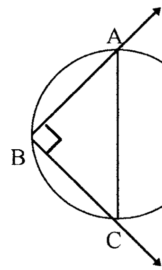
∠ABC is an inscribed angle since its vertex is on arc ABC. ∠ABC intercepts arc AC.

Measure of an inscribed angle: The measure of an inscribed angle is one-half of the measure of its intercepted arc.

Given: Arc AC measures 150°

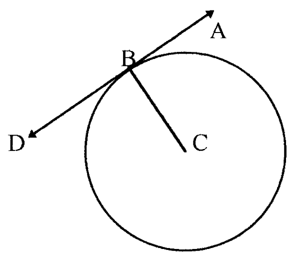


Since the measure of arc AC is 150° the measure of ∠ ABC is 75°.



Since the measure of ∠ ABC is 90°, the measure of arc AC is 180°. This means AC is a diameter of the circle.

Tangent Line and Radius of a Circle: A tangent line to a circle is perpendicular to the radius of a circle drawn to the point of tangency.

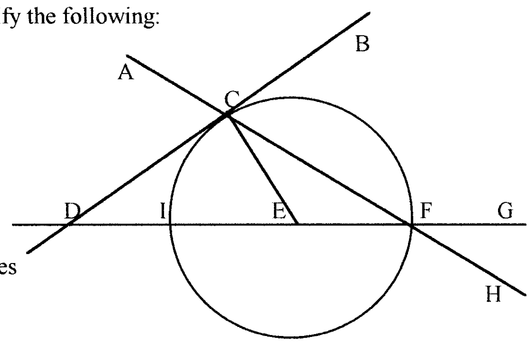


AD ⊥ BC, so ∠ ABC and ∠ DBC are right angles

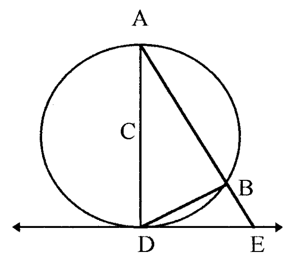
Assignment #12

From the figure at the right, identify the following:
Point E is the center of the circle.

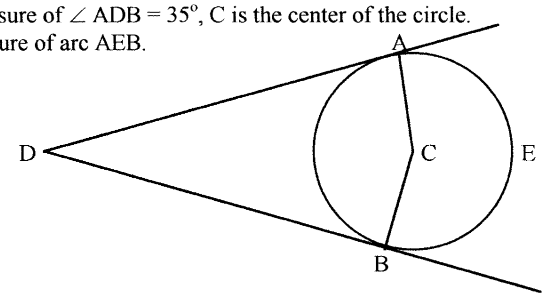
1.
   a) A secant line
   b) A tangent line
   c) A central angle
   d) An inscribed angle
   e) A minor arc
   f) A major arc
   g) Two perpendicular lines
   h) A chord



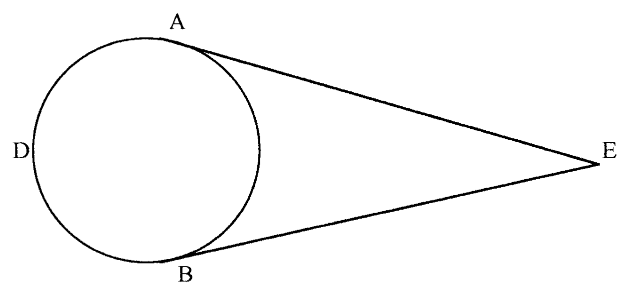
2. Given $\angle ADB = 50^{o}$, $\overline{AD}$ is a diameter.
   Find:
   a) Measure of arc AB
   b) Measure of $\angle$ ABD
   c) Measure of $\angle$ BAD
   d) Measure of arc ADB
   e) Measure of $\angle$ ADE
   f) Measure of $\angle$ BED



3. Given the measure of $\angle ADB = 35^{o}$, C is the center of the circle.
   Find the measure of arc AEB.



4. Given the measure of arc ADB = 200^{o}
   Find the measure of $\angle$ AEB



Length of Arc of a Circle

We have found that the measure of an arc is equal to the measure of its central angle and the measure of entire circle is 360^{o}. The ratio of the length of arc of a circle to the circumference of a circle is the same as the ratio of the measure of the central angle of the arc to 360^{o}. Symbolically, this is given by $\frac{L}{2\Pi r} = \frac{n}{360}$ where L represents the length of arc, n is the measure of the central angle of the arc in degrees and r is the radius of the circle. Solving for L gives: $L = \frac{n\Pi r}{180}$.

Example 1: Find the length of arc given a radius of 5 inches and a central angle of 30^{o}.

Solution: $L = \frac{n\Pi r}{180}$, $r = 5$, $n = 30$. Substituting for the appropriate variables gives:

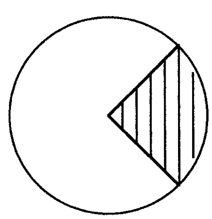
$L = \frac{(30)(\Pi)(5)}{180}$. Simplifying the right side of the equation gives $L = \frac{5\Pi}{6}$ inches.
Thus $L \approx 2.6$ inches.

Example 2: Find the measure of the central angle given the length of arc is 22 cm. and the radius is 10 cm.

Solution: $L = \frac{n\Pi r}{180}$, $L = 22$, $r = 10$. Substituting for the appropriate variables gives:
$22 = \frac{n\Pi(10)}{180}$. Solving for n gives $\frac{(22)(180)}{(10)(\Pi)} = n$ or $n = \frac{396}{\Pi}$. Thus $n \approx 126^{o}$.

Area of a Sector of a Circle

The sector of a circle is the region bounded by an arc and two radii of the circle.



The ratio of the area of a sector to the area of the circle is equal to the ratio of the measure of the central angle of the arc created by the two radii of the sector to 360°. Symbolically, this is given by $\frac{A}{\Pi r^2} = \frac{n}{360}$, where A represents the area of the sector, n represents the measure of the central angle of the arc in degrees and r represents the length of the radius of the circle. Solving for A gives A = $\frac{n\Pi r^2}{360}$.

Example #3: Find the area of a sector given its radius is 10 inches and the measure of its central angle is 130°.

Solution: Since A = $\frac{n\Pi r^2}{360}$, substituting 130 for n and 10 for r yields A = $\frac{130\Pi(10)^2}{360}$. Evaluating the right side of the equation gives A = $\frac{325}{9}\Pi$ square inches or approximately 113.45 square inches.

Example #4: Find the radius of a sector given its central angle is 75° and the area of the sector is 235 square centimeters.

Solution: Since A = $\frac{n\Pi r^2}{360}$, substituting 75 for n and 235 for A yields 235 = $\frac{75\Pi(r)^2}{360}$. By multiplying each side of the equation by 360 gives (235)(360) = 75 Πr^2. Dividing each side of the equation by 75 Π gives $\frac{(235)(360)}{75\Pi} = r^2$. Taking the square root of each side of the equation solves for r yielding $\sqrt{\frac{(235)(360)}{75\Pi}} = r$ or r is approximately 18.95 centimeters.

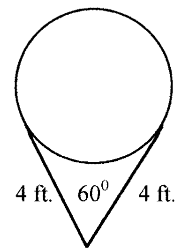
Assignment #13

1. Determine the lengths of arc from the given information:
   a) r = 12 in. ; central angle = 25° b) d = 25 m. ; central angle = 130°

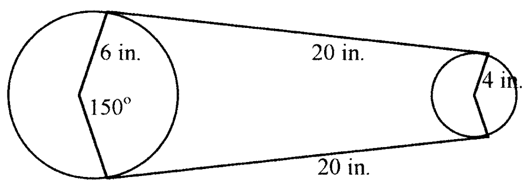
2. Find the measure of the central angle of an arc given the length of the arc is 28 in. and the radius measures 17 in.

3. Find the radius of an arc given the length of the arc is 324 cm. and the central angle measures 95°.

4. Find the perimeter of the figure to the right. The line segments are tangent to the arc.



5. Find the length of the pulley belt needed to reach around the two pulleys.

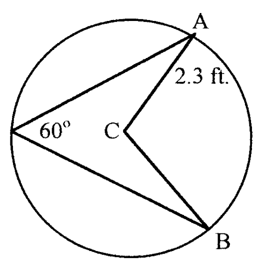


6. Determine the area of the sectors from the given information:
   a) r = 12 in. ; central angle = 25°   b) d = 25 m. ; central angle = 130°

7. Find the central angle of the sector given the area of the sector is 22.5 square centimeters and the radius is 4.2 centimeters.

8. Find the radius of the sector given the area of the sector is 7.3 square miles and the central angle measures 140°.

9. Find the area of sector created by the central angle ACB, given AC = 2.3 feet.



10. Find the area of the following region. The region was created by removing circular sectors from a regular hexagon. The dotted lines are on the hexagon and intersect at the centers of the circles.