Sequences

Name:____________________    Worked With:_________________________________

Instructions: It is expected that you have watched the section video and read the book prior to class. If you are asked to work in a group, please take a few minutes to introduce yourselves. Please answer each question as completely as possible. Have fun!

A sequence is generally thought of as any **ordered** list. A cookie recipe, for example, could be viewed as sequence because each step must be followed in the **order** given. If you bake the ingredients first, instead of last, you've ruined everything.

DNA is another sequence—change or mutate the **order** and you end up with a genetic **disorder** of some kind.

1. List at least 3 other real-life sequences.

In math, the only way to impose **order** on a function is to control its inputs. So, a mathematical sequence is any function whose domain is the natural numbers 1, 2, 3, 4, ... . To make the distinction between a function and a sequence clear we use \( n \) (a generic natural number) instead of \( x \) (a generic real number) as the input variable in sequences and name the sequence \( a_n \) instead of \( f(x) \).

2. Discuss the differences between the function \( f(x) = 2x + 3 \) and the sequence \( a_n = 2n + 3 \).

3. What is the 85\(^{th} \) term of the sequence \( a_n = 2n - 3(n - 1) \)?

4. Find \( a_{57} \) if \( a_n = \frac{4}{n} - n^2 \).

5. Find \( a_5 \) if \( a_n = a_{n-1} + 2 \) and \( a_1 = 3 \). [Hint: This is a recursive sequence.]

When we plot sequences we don’t get a nice, continuous curve—just a bunch of dots.

6. Plot at least 4 terms of the **arithmetic sequence** \( a_n = 4 + 2(n - 1) \). Then think about these questions:
   • What type of function does this remind you of?
   • Where does the sequence start?
   • In which direction is it headed?
   • Will it ever go the other way?
   • Can you make more dots in-between the points you graphed?
7. Plot at least 4 terms of the geometric sequence \( a_n = 1 \cdot 2^{n-1} \).

Then think about these questions:
- What type of function does this remind you of?
- Where does the sequence start?
- In which direction is it headed?
- Will it ever go the other way?

8. When a sequence contains \((-1)^n\) as a factor we call it an alternating sequence. Make a graph of \( a_n = (-1)^n \left( \frac{1}{n} \right) \) and see if you can notice this alternating behavior.

9. List at least 2 different ways to tell if a sequence is arithmetic or geometric.

10. Decide if the sequence \(-4, -1, 2, 5, 8, \ldots\) is arithmetic or geometric. Then give it's formula.

11. Decide if the sequence \(1, -3, 9, -27, 81, -243\) is arithmetic or geometric. Then give it's formula.

12. Decide if the sequence \(5, 2, 1, \frac{1}{2}, \frac{1}{4}\) is arithmetic or geometric. Then give it's formula.

13. Suppose a house, originally purchased in 1963 for $12,000, increases in value 10% per year.
   (a) Write a sequence which describes the value of the home.

   (b) What is that house worth today?

14. Suppose you bought a $1500 computer and that its value is dropping by $450 dollars per year. How much will it be worth in 3 years?
   (a) Write a sequence describing the value of the computer.

   (b) What is the computer worth today?