Transformations of Graphs (key)

1. (a) Shifts are caused by addition/subtraction  (b) Stretches are caused by multiplication/division
   (c) The constant is outside the function  (d) The constant is inside the function

2. (a) $x^2 + 4$  (b) $(x + 2)^2$
   (c) $2x^2$  (d) $(3x)^2$

3. (a) Achilles’ Ankle reaches its maximum in 1/10th the time
   (b) Baby Ben goes 1/10th as high
   (c) Catapult Charlie goes 10 times as high
   (d) Diablo Durmient ignites 10 seconds later
   (e) Elevated Ed is shot of a 10 foot high platform

4. $f(2x)$ will only go 1/2 as far (it still has the same height, though) and would not be a better driver. To double the distance the transformation should be $f\left(\frac{x}{2}\right)$. Using $2f(x)$ doubles the height but does not change the distance.

5. (a) $P(h) + 100$ would be getting an extra $100.
   (b) Shifting down would mean being fined a fixed amount while shifting up would be getting a fixed bonus.
   (c) Shifting LEFT actually makes more money, since you would be getting paid for 10 extra hours.

6. a 2 second head start is a shift left 2 spaces
   a 2 second penalty is a shift right 2 spaces
7. (a) \[-f(x) = -(2x + 3)^2 + 9\] and \[f(-x) = -(2x + 3)^2 - 9\]

(b) The values for \(-f(x)\) are simply the negatives of the values for \(f(x)\).

The values for \(f(-x)\) are the same values for \(f(x)\) but in the reverse order.

<table>
<thead>
<tr>
<th>(x)</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(x))</td>
<td>0</td>
<td>-8</td>
<td>-8</td>
<td>16</td>
<td>40</td>
<td>72</td>
<td></td>
</tr>
<tr>
<td>(-f(x))</td>
<td>0</td>
<td>8</td>
<td>8</td>
<td>0</td>
<td>-16</td>
<td>-40</td>
<td>-72</td>
</tr>
<tr>
<td>(f(-x))</td>
<td>72</td>
<td>40</td>
<td>16</td>
<td>0</td>
<td>-8</td>
<td>-8</td>
<td>0</td>
</tr>
</tbody>
</table>

(c) The vertical reflection is mirrored across the x-axis. The horizontal reflection is mirrored across the y-axis. If you happen to have a small mirror you can actually put in on the graph to see the reflections.

8. \[f(x)\] \[-f(x)\] \[f(-x)\]