LC Filters, Twin-T Filters, and Wien-Bridge Filters
Objectives

- Analyze the operation of LC low-pass filters
- Analyze the operation of LC high-pass filters
- Recognize common LC filter configurations, Twin-T filters, and Wien-Bridge filters
LC Low-Pass Filters

- Recall the Series Resonant RLC Circuit (sometimes called the Series Resonant LC Circuit).

- Note that if the output is taken across the capacitor, and the resistor is the source resistance, the output will be a low pass filter, and the resonant frequency will also be the high cutoff frequency.

- At resonance recall that $V_{OUT} = V_C = Q \times V_{IN}$
LC Low-Pass Filters

- In order for the output voltage to not rise sharply at resonance, the value of $R_L$ is chosen to reduce the circuit $Q$ to approximately 1. Since $V_{OUT} = V_C = Q \times V_{IN}$ at resonance, $Q$ must be 1 to make $V_{OUT} = V_{IN}$.

- When a Series Resonant RLC circuit is designed for a $Q \leq 1$, it is often referred to as an LC filter, although both names are often used interchangeable.

- The Frequency Response will be much sharper than for RLC and for either RC or RL LP Filters. (-40dB/decade – 2-pole)

$L = RL/2\pi f_C$ and $C = 1/(2\pi f_C RL)$ assuming $RS = RL$. 

![Inverted L Low Pass LC Filter](image)
LC Low-Pass Filters

- Usually, LC Filters are used in situations requiring impedance matching between RS and RL for communications circuitry, for example (600 ohms) and for loudspeaker systems (2-16 ohms).
- Common LC LP configurations for impedance matching are the 3rd-order T or PI section filters.
- These types of filters are sometimes referred to as ‘constant-k’ filters because the ratio of XL to XC is constant within the passband.

![PI-Section Low Pass LC Filter](image)

![TEE-Section Low Pass LC Filter](image)
LC Low-Pass Filters

- $Z_C = Z_0 \sqrt{1 - (f/f_C)^2}$, characteristic impedance of filter (actual value).
- $Z_{O(LC)} = k = \sqrt{L/C}$ = characteristic impedance of filter.
- $Z_{O(LC)}$ is used to describe the characteristic impedance of the filter and is equal to the load impedance $Z_O$. Ideally, it is constant within the passband of the filter assuming $Z_{IN} = Z_O$ (or $RS = RL$).
- To match $RS$ ($Z_{IN}$) to $RL$ ($Z_O$) for maximum power transfer, $Z_{O(LC)}$ appears as $Z_O$ to $Z_{IN}$ and $Z_{O(LC)}$ appears as $Z_{IN}$ to $Z_O$ within the passband.
- $f_C = 1/(\pi \sqrt{LC})$.
- Usually, the $f_C$ and the $RS$ and $RL$ are specified, so the values for $L$ and $C$ may be calculated as follows: $L = RL/\pi f_C$ and $C = 1/(\pi f_C RL)$. 
LC High-Pass Filters

• Again, recall the Series Resonant RLC Circuit.

• Note that if the output is taken across the inductor, and the resistor is the source resistance, the output will be a high pass filter, and the resonant frequency will also be the low cutoff frequency.

• At resonance recall that $V_{\text{OUT}} = V_L = Q \times V_{\text{IN}}$
LC High-Pass Filters

- In order for the output voltage to not rise sharply at resonance, the value of $R_L$ is chosen to reduce the circuit Q to approximately 1. Since $V_{OUT} = V_L = Q \times V_{IN}$ at resonance, Q must be 1 to make $V_{OUT} = V_{IN}$.
- When a Series Resonant RLC circuit is designed for a $Q \leq 1$, it is referred to as an LC filter.
- The Frequency Response will be much sharper than for RLC and for either RC or RL HP Filters. (-40dB/decade – 2-pole)

\[ L = \frac{R_L}{2\pi f_C} \text{ and } C = \frac{1}{(2\pi f_C R_L)} \text{ assuming } RS = RL. \]
LC High-Pass Filters

- Usually, LC Filters are used in situations requiring impedance matching between RS and RL for communications circuitry, for example (600 ohms) and for loudspeaker systems (2-16 ohms).
- Common LC HP configurations for impedance matching are the 3\textsuperscript{rd}-order T or PI section filters.
- These types of filters are sometimes referred to as ‘constant-k’ filters because the ratio of XL to XC is constant within the passband.

![TEE-Section High Pass LC Filter](image1)
![PI-Section High Pass LC Filter](image2)
LC High-Pass Filters

• \( Z_C = Z_O \sqrt{1 - \left(\frac{f_C}{f}\right)^2} \), characteristic impedance of filter (actual value).
• \( Z_{O(LC)} = k = \sqrt{\frac{L}{C}} \) = characteristic impedance of filter.
• \( Z_{O(LC)} \) is used to describe the characteristic impedance of the filter and is equal to the load impedance \( Z_O \). Ideally, it is constant within the passband of the filter assuming \( Z_{IN} = Z_O \) (or \( RS = RL \)).
• To match \( RS \) (\( Z_{IN} \)) to \( RL \) (\( Z_O \)) for maximum power transfer, \( Z_{O(LC)} \) appears as \( Z_O \) to \( Z_{IN} \) and \( Z_{O(LC)} \) appears as \( Z_{IN} \) to \( Z_O \) within the passband.
• \( f_C = 1/(4\pi\sqrt{LC}) \).
• Usually, the \( f_C \) and the \( RS \) and \( RL \) are specified, so the values for \( L \) and \( C \) may be calculated as follows: \( L = RL/4\pi f_C \) and \( C = 1/(4\pi f_C RL) \).
LC Filter Example 1

- Calculate the values for L, C, and RL if RS = 75 ohms and we wish maximum power transfer when $f_C = 10$ KHz.

- Is this filter predominately capacitive or inductive at frequencies below $f_C$?

- For maximum power transfer, $RL = RS = 75$ ohms.

- At $f_{CL} = 10$ KHz, $L = RL/(4\pi f_C) = 75\Omega/(4\pi(10\text{KHz})) = 597\mu\text{H}$.

- At $f_{CL} = 10$ KHz, $C = 1/(4\pi(10\text{KHz})(75\Omega)) = 0.106\mu\text{F}$;
  - $C_1 = C_2 = 2C = 2(0.106\mu\text{F}) = 0.212\mu\text{F}$

- At frequencies below $f_C$, $X_C$ increases, $X_L$ decreases. Therefore, more of the input voltage is dropped across $C$ and less across $L$. The filter becomes more capacitive at frequencies below $f_C$. 
Twin T-Filter

• The Twin T-filter is a 2-pole filter configuration with a standard -40dB/decade roll-off. The current in the resistors is in phase with the voltage. The phase shift of the current in the capacitors will be 180 degrees. At a certain frequency the current amplitudes will be the same, but in anti-phase. This frequency will therefore be suppressed. The circuit is therefore called a band stop filter, or notch filter. The center frequency equals:

\[ f_c = \frac{1}{2 \cdot \pi \cdot R \cdot C} \]

\[ R_1 = R_2 = R \]
\[ C_1 = C_2 = C \]
\[ R_3 = R/2 \]
\[ C_3 = 2C \]
\[ f_0 = 1/(2\pi RC) \]
Twin T-Filter

-40dB/decade or -12dB/octave roll-off
Bridged T-Filters

- These are also band-stop filters. Both filters have the same frequency response.
- Bridged T-filters have adjustable roll-off rates determined by the circuit Q. Higher Q’s produce notch filters with roll-off rates even steeper than that of a twin T-filter.
- To calculate the central frequency, we can use the same equation as for simple LC filters:

\[ f = \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}} \]

- At this frequency the output voltage will be 0V if \( R = Q \cdot XL/4 \).
Bridged T-Filters
Wien-Bridge Filter

- The Wien-Bridge circuit consists of two filter networks: a low pass filter and a high pass filter connected in series and grounded. The two filter networks overlap at a single frequency that is attenuated.

All R’s are equal values.
All C’s are equal values.
Wien-Bridge Filter

- Wien-Bridge filters are 1-pole configurations with a -20dB/decade or -6dB/octave roll-off.
- Notch frequency is \( f_n = \frac{1}{2\pi RC} \).

\[
\text{mid frequency } f_m = \frac{1}{2\pi RC} \\
\text{gain } G = \frac{1 - \left(\frac{f}{f_m}\right)^2}{\sqrt{3(1 - \left(\frac{f}{f_m}\right)^2)^2 - 9 \left(\frac{f}{f_m}\right)^2}} \\
\varphi = \arctan \left( \frac{3 \times \frac{f}{f_m}}{\left(\frac{f}{f_m}\right)^2 - 1} \right)
\]

Bode Plot - Wien Bridge 1
\( R = 13.5 \, k\Omega, \, C = 0.10 \, \mu F \)

Gain (dB)

Frequency (Hz)